



U8434

Reg. No.:

Name:.....

**University of Kerala**

First Semester Degree Examination, November 2024

Four Year Under Graduate Programme

Discipline Specific Core Course

MATHEMATICS

UK1DSCMAT108- DIFFERENTIAL CALCULUS

Academic Level: 100-199

Time: 2 Hours**Max.Marks:56****Part A.****Answer All Questions, Objective Type. 1 Mark Each.****(Cognitive Level: Remember/Understand)****6 Marks. Time: 5 Minutes**

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
1.	At what points, if any, does the graph of $y = x^3 - 3x + 4$ have a horizontal tangent line?	Remember	CO1
2.	Find $\frac{d}{dx} [\tan(x^2 + 1)]$	Remember	CO1
3.	Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$	Understand	CO3
4.	Define exponential function with base b .	Understand	CO2
5.	Find x such that $\ln(x + 1) = 5$	Remember	CO2
6.	State extreme- value theorem.	Remember	CO3

Part B.**Answer All Questions , Two-Three sentences. 2 Marks Each.****(Cognitive Level: Remember/Understand/Apply)****10 Marks. Time: 20 Minutes**

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
7.	Show that $g(y) = \sqrt[3]{y-1}$ is the inverse of $f(x) = x^3 + 1$	Remember	CO1
8.	Find $\frac{ds}{dt}$ if $s = (1+t)\sqrt{t}$.	Remember	CO1
9.	Find the local linear approximation of $f(x) = \sin x$ at $x_0 = 0$.	Remember	CO2
10.	Use implicit differentiation to find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$	Understand	CO2
11.	Find the intervals on which $f(x) = x^2 - 2x$ is increasing and the intervals on which it is decreasing.	Apply	CO3

Part C.

Answer all 4 questions, choosing among options within each question.

Short Answer. 4 Marks Each.

(Cognitive Level: Remember/Understand/Apply/Analyse)

16 Marks. Time:35 Minutes

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
12.	A. Find $\lim_{x \rightarrow +\infty} \frac{3x+5}{6x-8}$ OR B. Evaluate $\lim_{x \rightarrow 0^+} x \ln x$	Understand	CO1 CO3
13.	A. Use logarithmic differentiation to find $\frac{d}{dx}[(x^2 + 1)^{\sin x}]$ OR B. Consider the function $f(x) = x^5 + x + 1$. Show that f has a differentiable inverse function. Use implicit differentiation to find a formula for the derivative of f^{-1} . Compute $(f^{-1})'(1)$.	Understand	CO2 CO1
14.	A. Let $y = \sqrt{x}$. Find formulas for Δy and dy . Evaluate Δy and dy at $x = 4$ with $dx = \Delta x = 3$. OR B. Find the two x -intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.	Analyse	CO2 CO3
15.	A. Solve $\frac{e^x - e^{-x}}{2} = 1$ for x . OR B. Consider the function $x^3 - 3x^2 + 1$. Use the first and second derivatives of f to determine the intervals on which f is increasing, decreasing, concave up, and concave down. Find all inflection points.	Apply	CO2 CO3

Part D.

Answer all 4 questions, choosing among options within each question.

Long Answer. 6 Marks Each.

(Cognitive Level: Understand/Apply/Analyse/Evaluate/Create) 24Marks. Time:60 Minutes

Qn. No.	Question	Cognitive Level	Course Outcome (CO)
16.	A. Draw the graph of $y = x $. Prove that $f(x) = x $ is not differentiable at $x = 0$ by showing that the limit in definition does not exist at $x = 0$. OR B. On a sunny day, a 50 ft flagpole casts a shadow that changes with the angle of elevation of the Sun. Let s be the length of the shadow and θ the angle of elevation of the Sun. Find the rate at which the length of the shadow is changing with respect to θ when $\theta = 45^\circ$. Express your answer in units of feet/degree.	Understand	CO1

17.	A. Discuss the limits $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$ and $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$. OR B. (i) What can you say about the continuity of the function $f(x) = \sqrt{9 - x^2}$? (ii) Show that the function x is continuous everywhere.	Understand	CO3 CO1
18.	A. A base ball diamond is a square whose sides are 90 ft long. Suppose that a player running from second base to third base has a speed of 30 ft/s at the instant when he is 20 ft from third base. At what rate is the player's distance from home plate changing at that instant? OR B. Suppose that the side of a square is measured with a ruler to be 10 inches with a measurement error of at most $\pm \frac{1}{32} \text{ in.}$ Estimate the error in the computed area of the square.	Analyse	CO2
19.	A. Find the relative extrema of $f(x) = 3x^5 - 5x^3$. OR B. Show that the function $f(x) = \frac{x^3}{4} + 1$ satisfies the hypotheses of the Mean-Value Theorem over the interval $[0, 2]$, and find all values of c in the interval $(0, 2)$ at which the tangent line to the graph of f is parallel to the secant line joining the points $(0, f(0))$ and $(2, f(2))$.	Apply	CO3